

ECS315 2014 Quiz 4 Solution

Continue from Quiz 3 and Ex. 8.13

Suppose a RV X has pmf $p_X(x) = \begin{cases} \frac{6}{11x}, & x=1,2,3, \\ 0, & \text{otherwise.} \end{cases}$

① Find $\mathbb{E}X = \sum_x x p_X(x) = \sum_x x \frac{6}{11x} = 3 \times \frac{6}{11} = \frac{18}{11}$

There are three values in the support of X . Therefore, the sum here has three x -values.

② Suppose $Y = (X-2)^2$. a) Find $p_Y(y)$

x	$y = (x-2)^2$
1	1
2	0
3	1

$P[Y=0] = P[X=2] = \frac{6}{11 \times 2} = \frac{3}{11}$

$P[Y=1] = P[X=1] + P[X=3] = 1 - \frac{3}{11} = \frac{8}{11}$

$p_Y(y) = \begin{cases} 3/11, & y=0, \\ 8/11, & y=1, \\ 0, & \text{otherwise} \end{cases}$

b) Find $\mathbb{E}Y$ from $p_Y(y)$

$= 0 \times \frac{3}{11} + 1 \times \frac{8}{11} = \frac{8}{11}$

c) Find $\mathbb{E}Y$ via LOTUS

$= \sum_x g(x) p_X(x) = \sum_x (x-2)^2 p_X(x)$

$= 1 \times \frac{6}{11} + 0 \times \frac{3}{11} + 1 \times \frac{2}{11} = \frac{8}{11}$

③ Suppose $Z = Y + 3(X-2) + 1$. Find $\mathbb{E}Z$.

$\mathbb{E}Z = \mathbb{E}[Y + 3(X-2) + 1] = \underbrace{\mathbb{E}Y}_{\frac{8}{11}} + \underbrace{\mathbb{E}[3X-6]}_{3\mathbb{E}X-6} + \underbrace{\mathbb{E}[1]}_1 = \frac{7}{11}$

$3 \times \frac{18}{11} - 6$

Alternatively,

$p_X(x)$	x	$y = (x-2)^2$	$z = y + 3(x-2) + 1$
6/11	1	1	$1 + 3(-1) + 1 = -1$
3/11	2	0	$0 + 3(0) + 1 = 1$
2/11	3	1	$1 + 3(1) + 1 = 5$

$p_Z(z) = \begin{cases} 6/11, & z = -1, \\ 3/11, & z = 1, \\ 2/11, & z = 5, \\ 0, & \text{otherwise.} \end{cases}$

Therefore, $\mathbb{E}Z = \frac{6}{11} \times (-1) + \frac{3}{11} \times (1) + \frac{2}{11} \times 5 = \frac{-6+3+10}{11} = \frac{7}{11}$